

# $1/m_Q$ and $1/N_c$ Expansions for Excited Heavy Baryons with Light Quarks in the Spin-Flavor Symmetric Representation

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## Abstract

The mass spectrum of the  $L = 1$  orbitally excited heavy baryons with light quarks in the spin-flavor symmetric representation is studied by the  $1/N_c$  expansion method in the framework of the heavy quark effective theory. The mixing effect from the baryons in the mixed representation is considered. The general pattern of the spectrum is predicted which will be verified by the experiments in the near future. The  $1/m_Q$  and  $SU(3)$  corrections are also considered. Mass relations for the baryons  $\Lambda_{c1}^{(*)}$ ,  $\Sigma_{c1}^{(*)}$ ,  $\Xi_{c1}^{(')(*)}$ , and  $\Omega_{c1}^{(*)}$  are derived.

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A lot of data for orbitally excited heavy baryons have been accumulating experimentally [1,2]. Understanding them will extend our ability in the application of QCD. The heavy quark effective theory (HQET) [3] provides a systematic way to investigate hadrons containing a single heavy quark. To obtain detailed prediction, however, some nonperturbative QCD methods have to be used. In this paper,  $1/N_c$  expansion [4] is applied in the analysis. Within this framework, the masses of  $L = 1$  orbitally excited heavy baryons with light quarks in both the spin-flavor symmetric and mixed representation have been analyzed [5]. By the HQET sum rule, masses of lowest state of the excited baryons have also been calculated [6,7]. They were studied in other approaches, too, for example in quark models [8], in the chiral Lagrangian formalism [9] and in the Skyrme model or the large  $N_c$ <sup>1</sup> HQET [10]. In constituent quark models [8], the classification of the baryons according to the light quark spin-flavor symmetry is taken to be physical. In the treatment of the baryons with light quarks in the spin-flavor symmetric representation in Ref. [5], it was erroneous to take only one light quark being excited. In fact, it is the heavy quark that is orbitally excited. Note that the orbital excitation of the heavy quark is not suppressed by the mass of the heavy quark. Or relatively speaking, it is the light quark pair as a whole in which the two light quarks have zero relative orbital angular momentum, that is  $L = 1$  excited [6–9]. This paper reconsiders the excited heavy baryons with light quarks in the spin-flavor symmetric representation in the approach of  $1/N_c$  expansion within the framework of the HQET. The results are very simple. Furthermore, the mixing between the two kinds of representations will also be discussed by  $1/N_c$  expansion, which is argued being small. Therefore our results are physical and predictive.

In the HQET, many features of heavy hadrons have been analyzed. In the heavy quark

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<sup>1</sup>We make distinction between  $1/N_c$  expansion and the large  $N_c$  limit. Because the former is essentially based on the light quark spin-flavor symmetry in the baryon sector, the leading order result of it is not that of  $N_c \rightarrow \infty$ . See Manohar, in Ref. [4].

limit, the heavy quark spin decouples from the strong interaction. The mass of a heavy hadron  $H$  is expanded as

$$M_H = m_Q + \bar{\Lambda}_H - \frac{\lambda_1^H}{2m_Q} + \frac{\lambda_2^H}{2m_Q} + O\left(\frac{1}{m_Q^2}\right), \quad (1)$$

where  $m_Q$  is the heavy quark mass, the parameter  $\bar{\Lambda}_H$  is independent of the heavy quark spin and flavor, and describes mainly the contribution of the light degrees of freedom in the baryon.  $\lambda_1^H$  and  $\lambda_2^H$  are the kinetic and chromomagnetic matrix elements, respectively,

$$\begin{aligned} \lambda_1^H &= \langle H(v) | \bar{h}_v (iD)^2 h_v | H(v) \rangle, \\ \lambda_2^H &= -\langle H(v) | \bar{h}_v \frac{g_s}{2} G_{\mu\nu} \sigma^{\mu\nu} h_v | H(v) \rangle, \end{aligned} \quad (2)$$

with  $h_v$  denoting the heavy quark field with velocity  $v$ . The quantities  $\bar{\Lambda}_H$ ,  $\lambda_1^H$  and  $\lambda_2^H$  should be calculated by nonperturbative HQET.

At this stage, the  $1/N_c$  expansion is applied in the analysis. It is one of the most important and model-independent methods of nonperturbative QCD. Nonperturbative properties of mesons can be observed from the analysis of the planar diagrams, and baryons from the Hartree-Fock picture. For the ground state baryons, it has been found that there is a contracted  $SU(2N_f)$  light quark spin-flavor symmetry in the large  $N_c$  limit [11–14]. This makes a  $1/N_c$  expansion based on the spin-flavor structure possible for the baryons. Many quantitative predictions and further extensions of the above result have been made [15–20].

Before we go on, two remarks should be made. First, the above mentioned  $1/N_c$  expansion applies to the s- or p-wave states of low spin in the baryon multiplet. The states with spin of order  $N_c/2$  are considerably modified by spin-spin and spin-orbit interactions [12]. Second, It is actually  $N_c - 1$ , which is 2 in real World, that will be taken as a large number, because heavy quark is distinguished. This is an improvement compared to the  $1/N_c$  for the excited heavy baryons with light quarks in the mixed representation. In that case, the expansion parameter is  $N_c - 2$  [5].

The quantum numbers which describe the hadrons are angular momentum  $J$  and isospin  $I$ . For the heavy hadrons, the total angular momentum of the light degrees of freedom  $J^l$

becomes a good quantum number in the HQET. In the light quark spin-flavor symmetric representation, the light degrees of freedom in  $H$  look like a collection of  $N_c - 1$  light quarks without orbital angular momentum excitation. This picture for the light quarks is essentially the same as that of the ground state heavy baryons. The spin-flavor decomposition rule is  $I = S^l$  for the non-strange baryons, where  $S^l$  is the total spin of the light quark system. Note that the light quark system as a whole has  $L = 1$  orbital angular momentum. In other words, the heavy quark now is  $L = 1$  excited in this case. In real World  $N_c$  is fixed to be 3, so there are only two light quarks in the heavy baryon. The spin-flavor structure of them is quite simple,  $(I, S^l) = (0, 0)$  and  $(1, 1)$ . All possible states of excited heavy baryons are listed in Table I. In the table, except the third state, the other six states form three pairs. Each pair is a doublet under the heavy quark spin symmetry. We adopt the Hartree-Fock picture to study  $\bar{\Lambda}_H$  where in the baryon  $H$ , the light quarks are in the spin-flavor symmetric representation. One of the essential points of the  $1/N_c$  expansion is the  $N_c$  counting rules of the relevant Feynman diagrams.

In the Hartree-Fock picture of the baryons, the  $N_c$  counting rules require us to include many-body interactions in the analysis, instead of including only one- or two-body interactions. However, a large part of these interactions are spin-flavor irrelevant. Namely this part contributes in the order  $N_c \Lambda_{\text{QCD}}$  universally to all the baryons with different spin-flavor structure in Table I. This makes us arrive in an  $1/N_c$  expansion based on the light quark spin-flavor structure of the baryons. The mass splittings among the baryons in the same light quark spin-flavor representation can be obtained. For the purely light quark contribution to  $\bar{\Lambda}_H$ , the  $1/N_c$  analysis goes the same as that to the ground state heavy baryons [12]. There is a light quark spin-flavor symmetry at the leading order of the  $1/N_c$  expansion.  $\bar{\Lambda}_H$  is trivially  $\sim N_c \Lambda_{\text{QCD}}$  at this order. The mass splitting due to the light quark spin-flavor symmetry violation started from  $S^{l^2}/N_c$  [12]. However, different from the ground state baryons, formally the orbital angular momentum of the heavy quark has more dominant contribution to  $\bar{\Lambda}_H$  than  $O(1/N_c)$ . This is because of the orbital-light-quark-spin interactions. After summing up all the relevant many-body interactions, this order  $O(1)$

contribution is  $\vec{L} \cdot \vec{S}^l f(\frac{S^{l^2}}{N_c^2})$ , where  $f$  is a general function which can be Taylor expanded. The mass  $\bar{\Lambda}_H$  can be written simply as

$$\bar{\Lambda}_H^0 = N_c \tilde{c}_0 + \tilde{c}_1 \vec{L} \cdot \vec{S}^l + \tilde{c}_2 \frac{S^{l^2}}{N_c} + O\left(\frac{1}{N_c^2}\right), \quad (3)$$

where coefficients  $\tilde{c}_i \sim \Lambda_{\text{QCD}}$  ( $i = 0, 1, 2$ ). There should be also term proportional to  $L^2$  in the above equation, which gives constant contribution to  $\bar{\Lambda}_H^0$  for a given light quark representation, and therefore has been absorbed into the leading term. The term  $\vec{L} \cdot \vec{S}^l$  can be rewritten as  $J^{l^2} - S^{l^2}$  with  $\vec{J}^l$  being defined as  $\vec{J}^l = \vec{L} + \vec{S}^l$ . Therefore

$$\bar{\Lambda}_H^0 = N_c c_0 + c_1 (J^{l^2} - S^{l^2}) + c_2 \frac{S^{l^2}}{N_c} + O\left(\frac{1}{N_c^2}\right), \quad (4)$$

where coefficients  $c_i \sim \Lambda_{\text{QCD}}$  need to be determined from experiments.

The numerical results are also given on the right-handed side of Table I. Because the mass formula of Eq. (4) is rather simple, some features of the spectrum can still be discussed. The parameters  $c_0$  and  $c_2$  are naturally expected to be positive. However  $c_1$  can have both signs. If  $c_1 > 0$ , we see that the singlet state  $(J, I) = (\frac{1}{2}, 1)$  could be the lowest state. By requiring the first doublet to be the lowest, we must have  $c_2 > 2N_c c_1$ . The resulting spectrum will be

$$M(\frac{1}{2}(\frac{3}{2}), 0, 1, 0) < M(\frac{1}{2}, 1, 0, 1) < M(\frac{1}{2}(\frac{3}{2}), 1, 1, 1) < M(\frac{3}{2}(\frac{5}{2}), 1, 2, 1) \quad (5)$$

with the quantum numbers denoting  $J$ ,  $I$ ,  $J^l$  and  $S^l$ , respectively. On the other hand, if  $c_1 < 0$ , the first doublet is the lowest states only if  $c_2 > -N_c c_1$ . In this case, the singlet is the heaviest, and the spectrum is

$$M(\frac{1}{2}(\frac{3}{2}), 0, 1, 0) < M(\frac{3}{2}(\frac{5}{2}), 1, 2, 1) < M(\frac{1}{2}(\frac{3}{2}), 1, 1, 1) < M(\frac{1}{2}, 1, 0, 1). \quad (6)$$

None of the above discussed spectrum pattern is consistent with the quark model prediction [8]. It should be noted that our analysis neglected the  $1/N_c^3$  correction (compared to the leading order) which is expected to be not significant.

The conditions for  $c_2$  are not satisfactory, although they are not unreasonable considering that in real World  $N_c$  is not large. In fact, this unsatisfactory point can be avoided if the mixing effect from the baryons in the mixed representation is considered.

It is necessary to consider the mixing between the baryons with light quarks in the spin-flavor symmetric and mixed representations. When they have same quantum numbers of  $(J, I, J^I)$ , there is no physical way to distinguish them. This consideration will give the physical spectrum. Because of the light quark spin-flavor symmetry at the leading order of  $1/N_c$  expansion, the baryons with same  $(J, I, J^I)$  quantum numbers but in different representations do not mix. The mixing occurs at the sub-leading order. The classification of baryons by the spin-flavor symmetry is therefore physical at the leading order [21]. For the physical spectrum, the mixing results in a deviation from  $\bar{\Lambda}_H^0$ . By denoting the mixing mass as  $\tilde{m}$  which is of  $O(1)$ , the mass matrix for the baryons with same  $(J, I, J^I)$  is written as

$$\begin{pmatrix} \bar{\Lambda}_H^0 & \tilde{m} \\ \tilde{m} & \bar{\Lambda}_{H'}^0 \end{pmatrix}, \quad (7)$$

where  $H'$  is the corresponding baryon in the mixed representation.  $\bar{\Lambda}_{H'}^0$  was given in Ref. [5]. The mass difference  $\bar{\Lambda}_H^0 - \bar{\Lambda}_{H'}^0$  is  $O(1)$ . Taking  $\tilde{m} < \bar{\Lambda}_{H'}^0 - \bar{\Lambda}_H^0$  for illustration, the physical mass are corrected to be

$$\begin{aligned} \bar{\Lambda}_H &\simeq \bar{\Lambda}_H^0 - \frac{\tilde{m}^2}{\bar{\Lambda}_{H'}^0 - \bar{\Lambda}_H^0}, \\ \bar{\Lambda}_{H'} &\simeq \bar{\Lambda}_{H'}^0 + \frac{\tilde{m}^2}{\bar{\Lambda}_{H'}^0 - \bar{\Lambda}_H^0}. \end{aligned} \quad (8)$$

The mixing effect  $\frac{\tilde{m}^2}{\bar{\Lambda}_{H'}^0 - \bar{\Lambda}_H^0}$  is positive. It reduces the predictive power of Eq. (4) for the mass spectrum. The  $1/N_c$  expansion of  $\tilde{m}$  is parameterized as

$$\tilde{m} = \tilde{m}_0 + O(1/N_c), \quad (9)$$

where  $\tilde{m}_0$  is universal due to the light quark spin-flavor symmetry. To the order of  $O(1)$ , the spectrum is given as follows explicitly.

$$\begin{aligned}
\bar{\Lambda}_{(\frac{1}{2}(\frac{3}{2}),0,1)} &= N_c c_0 + 2c_1 - \frac{\tilde{m}_0^2}{k - c_{LS} - \frac{1}{6}\bar{c}_1 - \frac{1}{4}\bar{c}_2 - 2c_1} , \\
\bar{\Lambda}_{(\frac{1}{2},1,0)} &= N_c c_0 - 2c_1 , \\
\bar{\Lambda}_{(\frac{1}{2}(\frac{3}{2}),1,1)} &= N_c c_0 - \frac{\tilde{m}_0^2}{k} , \\
\bar{\Lambda}_{(\frac{3}{2}(\frac{5}{2}),1,2)} &= N_c c_0 + 4c_1 ,
\end{aligned} \tag{10}$$

where  $k$  is an  $O(1)$  constant that remains after the  $\bar{\Lambda}_{H'}^0$  and  $\bar{\Lambda}_H^0$  cancellation,  $\bar{\Lambda}_{H'}^0$  is parameterized by  $c_{LS}$ ,  $\bar{c}_1$  and  $\bar{c}_2$  which are around  $\Lambda_{\text{QCD}}$ , and can be found in the Table II of Ref. [5] (where  $\bar{c}_1$  and  $\bar{c}_2$  were denoted as  $c_1$  and  $c_2$ , respectively). Note that the masses of the states  $(\frac{1}{2}, 1, 0)$  and  $(\frac{3}{2}(\frac{5}{2}), 1, 2)$  are not affected by the mixing, because there are no physical states with the same good quantum numbers in the mixed representation. From the above spectrum, we see that  $c_1 > 0$ . The states  $(\frac{3}{2}(\frac{5}{2}), 1, 2)$  is always the highest states. They are heavier than the other states at least by  $4c_1$  through requiring the states  $(\frac{1}{2}(\frac{3}{2}), 0, 1)$  to be the lowest. If  $2c_1 > \frac{\tilde{m}_0^2}{k}$ , the requirement implies

$$\frac{\tilde{m}_0^2}{k - c_{LS} - \frac{1}{6}\bar{c}_1 - \frac{1}{4}\bar{c}_2 - 2c_1} > 4c_1 . \tag{11}$$

In this case, the spectrum pattern is

$$M(\frac{1}{2}(\frac{3}{2}), 0, 1) < M(\frac{1}{2}, 1, 0) < M(\frac{1}{2}(\frac{3}{2}), 1, 1) < M(\frac{3}{2}(\frac{5}{2}), 1, 2) . \tag{12}$$

On the other hand, if  $2c_1 < \frac{\tilde{m}_0^2}{k}$ , the requirement is

$$\tilde{m}_0^2 \left( \frac{1}{k - c_{LS} - \frac{1}{6}\bar{c}_1 - \frac{1}{4}\bar{c}_2 - 2c_1} - \frac{1}{k} \right) > 2c_1 , \tag{13}$$

which gives the spectrum

$$M(\frac{1}{2}(\frac{3}{2}), 0, 1) < M(\frac{1}{2}(\frac{3}{2}), 1, 1) < M(\frac{1}{2}, 1, 0) < M(\frac{3}{2}(\frac{5}{2}), 1, 2) . \tag{14}$$

Experimentally, the excited charmed baryons  $\Lambda_{c1}(\frac{1}{2})$  and  $\Lambda_{c1}(\frac{3}{2})$  have been found which correspond to the  $(\frac{1}{2}(\frac{3}{2}), 0, 1)$  states. More data are needed to fix the unknown parameters  $c_i$ 's,  $\bar{c}_i$ 's,  $k$  and  $c_{LS}$ . In the near future, experiments will check the above predicted spectrum.

Hopefully one of the above mass patterns will be picked out. It will be a check for the validity of our method, if the parameters are in the reasonable range ( $\Lambda_{\text{QCD}}$ ) and meanwhile satisfy the relations given above.

For a complete analysis of the heavy hadron masses,  $1/m_Q$  corrections have to be considered. The general expression of the corrections have been given in Eqs. (1) and (2). The quantities  $\lambda_1^H$  and  $\lambda_2^H$  can be analyzed by the  $1/N_c$  expansion in the similar way as  $\bar{\Lambda}_H$ . In the leading order of  $1/N_c$ ,  $\lambda_1^H$  is independent of the light quark structure and scales as unity. Therefore we have the following expansion ,

$$\begin{aligned}\lambda_1^H &= \tilde{c}_0' + \tilde{c}_1' \vec{L} \cdot \frac{\vec{S}^l}{N_c} + O\left(\frac{1}{N_c^2}\right) \\ &= c_0' + c_1' \frac{J^{l^2} - S^{l^2}}{N_c} + O\left(\frac{1}{N_c^2}\right).\end{aligned}\tag{15}$$

The mixing effect also affects  $\lambda_1^H$ . Its  $1/N_c$  expansion is that the non-vanishing contribution begins at  $O(1/N_c)$ . And at this order, the contribution is constant which can be absorbed into  $c_0'$ . The parameters  $c_0'/2m_Q$  and  $c_1'/2m_Q$  can be absorbed into  $c_0$  and  $c_1$  in Eq. (4), respectively. The inclusion of  $\lambda_1^H$  corrects the masses of the baryons at the order of  $1/m_Q$  which is expected to be not significant. It does not change the mass pattern given above to the order of  $O(1/m_c N_c)$ .

The degeneracy in the spectrum due to the heavy quark spin symmetry is lifted by  $\lambda_2^H$ . According to the definition in Eq. (2),  $\lambda_2^H$  is heavy baryon spin dependent. It is convenient to extract this dependence explicitly,

$$\lambda_2^H = d_H \lambda_2\tag{16}$$

where  $d_H = 2j^l$  for  $H$  with  $J = j^l + \frac{1}{2}$ , and  $d_H = -2j^l - 2$  for  $H$  with  $J = j^l - \frac{1}{2}$ . The new defined heavy quark hadronic matrix element  $\lambda_2$  is heavy baryon spin independent. It is also independent of the light quark structure and scales as unity in the leading order  $1/N_c$  expansion. Like  $\lambda_1^H$ , the  $1/N_c$  expansion for  $\lambda_2^H$  is

$$\lambda_2^H = d_H \left[ c_0'' + c_1'' \frac{J^{l^2} - S^{l^2}}{N_c} + O\left(\frac{1}{N_c^2}\right) \right].\tag{17}$$



The mixing effect for  $\lambda_2^H$  is that the leading nonzero contribution is  $O(1/N_c)$  which is constant and therefore can be absorbed into  $c_0$ ". The parameters  $c_i''$  should be determined by the experimental data. If we work to the accuracy of  $\Lambda_{\text{QCD}}/(m_Q N_c) \sim 10\%$ ,  $c_0''$  can be fixed from the mass splitting of  $\Lambda_{c1}(\frac{3}{2})$  and  $\Lambda_{c1}(\frac{1}{2})$ ,

$$c_0'' = \frac{m_c}{3} [M_{\Lambda_{c1}(\frac{3}{2})} - M_{\Lambda_{c1}(\frac{1}{2})}] \simeq (128 \text{ MeV})^2, \quad (18)$$

by taking  $m_c \simeq 1.5 \text{ GeV}$ . Note that  $c_0''$  is positive. The mass splittings of the other degenerate states listed in Table I are predicted to be

$$\begin{aligned} M(\frac{5}{2}, 1, 2, 1) - M(\frac{3}{2}, 1, 2, 1) &= \frac{5c_0''}{m_c} \simeq 55 \text{ MeV}, \\ M(\frac{3}{2}, 1, 1, 1) - M(\frac{1}{2}, 1, 1, 1) &= \frac{3c_0''}{m_c} \simeq 33 \text{ MeV}, \end{aligned} \quad (19)$$

to the accuracy of  $c_0''^2/(m_c N_c)$  which is about 5 MeV. These predictions can be checked with the experiments in the near future.

Finally, let us consider the case of the excited heavy baryons with light quarks including the strange quark. Very recently, there are experimental evidence of the charmed-strange analogs of  $\Lambda_{c1}(\frac{3}{2})$ ,  $\Xi_{c1}(\frac{3}{2})$  particles [2]. The above framework can be easily extended to include the charmed-strange baryons by taking strangeness as perturbation to the light quark flavor symmetry. The relevant baryon mass is then expressed as

$$M_H = m_Q + N_c c_0 + c_1(J'^2 - S'^2) + \text{mixing} + c_3(-s) + O\left(\frac{1}{N_c}\right), \quad (20)$$

where  $s$  is the heavy baryon strangeness number which can be 0,  $-1$ , or  $-2$ . The parameter  $c_3$  stands for the leading order of SU(3) correction to the  $\bar{\Lambda}_H$  given in Eq. (4). It is fixed by the mass difference of  $\Xi_{c1}(\frac{3}{2})$  and  $\Lambda_{c1}(\frac{3}{2})$ ,

$$c_3 \simeq 190 \text{ MeV}. \quad (21)$$

The mass of  $\Xi_{c1}(\frac{1}{2})$  is then predicted to be 190 MeV higher than  $\Lambda_{c1}(\frac{1}{2})$ ,

$$\begin{aligned} M_{\Xi_{c1}(\frac{1}{2})} &= M_{\Lambda_{c1}(\frac{1}{2})} + M_{\Xi_{c1}(\frac{3}{2})} - M_{\Lambda_{c1}(\frac{3}{2})} \\ &\simeq 2784 \text{ MeV}. \end{aligned} \quad (22)$$

Note that this prediction is only subject to a small uncertainty which is about  $c_3^2/(m_c N_c) \sim 10$  MeV. The future experiments may find that particles  $\Sigma_{c1}^{(*)}$ , which are the lowest charmed  $L = 1$  states with isospin 1, are just the state pair  $[\frac{1}{2}(\frac{3}{2}), 1, 1, 1]$  in Table I. Their strange analogs  $\Xi_{c1}^{\prime(*)}$  and  $\Omega_{c1}^{(*)}$  will then be predicted from the similar relation rather precisely,

$$\begin{aligned} M_{\Xi_{c1}^{\prime}(\frac{3}{2})} - M_{\Sigma_{c1}(\frac{3}{2})} &= M_{\Xi_{c1}^{\prime}(\frac{1}{2})} - M_{\Sigma_{c1}(\frac{1}{2})} + O\left(\frac{\Lambda_{\text{QCD}}}{m_c N_c}\right), \\ M_{\Omega_{c1}(\frac{3}{2})} - M_{\Sigma_{c1}(\frac{3}{2})} &= M_{\Omega_{c1}(\frac{1}{2})} - M_{\Sigma_{c1}(\frac{1}{2})} + O\left(\frac{\Lambda_{\text{QCD}}}{m_c N_c}\right). \end{aligned} \quad (23)$$

To the accuracy of  $s^2/N_c \sim 30\%$ ,

$$\begin{aligned} M_{\Omega_{c1}^{(*)}} - M_{\Xi_{c1}^{\prime(*)}} &= M_{\Xi_{c1}^{\prime(*)}} - M_{\Sigma_{c1}^{(*)}} + O\left(\frac{s^2}{N_c}\right) \\ &\simeq (190 \pm 70) \text{ MeV}. \end{aligned} \quad (24)$$

In summary, we have applied the  $1/N_c$  expansion method to study the mass spectrum of the  $L = 1$  orbitally excited heavy baryons with light quarks being in the spin-flavor symmetric representation within the framework of the HQET. The analysis is very simple compared to that for the heavy baryons with light quarks in the mixed representation in Ref. [5]. The simplicity is a unique feature in this case. It can be seen from the following point of view, namely the light quark system is in the ground state and it is the heavy quark that is orbitally excited. However the mixing effect due to the baryon states in the mixed representation corrects the spectrum pattern in the subleading order of  $1/N_c$  expansion. The effect is important to get the realistic spectrum at this order. The general pattern of the baryon spectrum has been given, which will be verified by the experiments in the near future. The  $1/m_Q$  and SU(3) corrections have also been considered. Certain mass relations for the baryons  $\Lambda_{c1}^{(*)}$ ,  $\Sigma_{c1}^{(*)}$ ,  $\Xi_{c1}^{\prime(*)}$ , and  $\Omega_{c1}^{(*)}$  have been derived. The same analysis can be applied to the bottom baryons.

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# TABLES

Table I. Excited heavy baryon states of the symmetric representation of  $N_c - 1$  light quarks. The masses are that without considering the mixing.

$(J, I)$	$(J^l, S^l)$	$\bar{\Lambda}_H$
$(1/2, 0)$	$(1, 0)$	$N_c c_0 + 2c_1$
$(3/2, 0)$	$(1, 0)$	$N_c c_0 + 2c_1$
$(1/2, 1)$	$(0, 1)$	$N_c c_0 - 2c_1 + \frac{2c_2}{N_c}$
$(1/2, 1)$	$(1, 1)$	$N_c c_0 + \frac{2c_2}{N_c}$
$(3/2, 1)$	$(1, 1)$	$N_c c_0 + \frac{2c_2}{N_c}$
$(3/2, 1)$	$(2, 1)$	$N_c c_0 + 4c_1 + \frac{2c_2}{N_c}$
$(5/2, 1)$	$(2, 1)$	$N_c c_0 + 4c_1 + \frac{2c_2}{N_c}$